Pop Quiz 8̶7, Chemistry 481, 6 Nov 2019.   Name: _____

SHOW YOUR WORK, YOU WILL NOT BE GIVEN FULL CREDIT IF YOU JUST CITE
RELEVANT EQUATIONS WITHOUT A BRIEF EXPLANATION OF THEIR ORIGIN

Possibly useful constants:

$h = 6.626 \times 10^{-34}$ J·s; c (speed of light) $= 2.998 \times 10^8$ m·s^{-1}

$k_B = 1.3807 \times 10^{-23}$ J; 1 amu $= 1.66 \times 10^{-27}$ kg

A monolayer of Ar is absorbed on a two-dimensional square surface. Assume the adsorbed gas
behaves like an ideal two-dimensional gas. For a two-dimensional-particle-in-a-box the partition
function is

$$q(T, A) = \left(\frac{2\pi m k_B T}{h^2} \right)^{2/2} A$$

where A is the area of the surface. Determine the change in entropy (ΔS) when n moles of a gas
adsorbed on a surface area of A are heated from T_1 to T_2 (A remains constant) (10 pts) We know
that $S = \langle E \rangle / T + k \ln Q$ and $Q = q^N / N!$ We also know that for an ideal gas $\langle E \rangle / T = (1/2)kT$
for each degree of freedom (here two degrees of freedom). Thus $\ln Q = N \ln q - N \ln N + N$ and
$\langle E \rangle / T = k$.

The change in entropy upon raising the temperature while keeping the surface area fixed
is

$$\Delta S = \int_{T_i}^{T_f} (\partial S / \partial T)_A dT = k \int_{T_i}^{T_f} \frac{\partial \ln Q}{\partial T}\bigg|_A dT$$

(since none of the other terms in S depends on T). From the expression for the single-particle
partition function, $\ln q = \ln T +$ other terms which are independent of T. Thus $(\partial \ln Q / \partial T)_A = N(\partial \ln q / \partial T)_A = N/T$ So, after integration we have

$$\Delta S = Nk \int_{T_i}^{T_f} \frac{\partial \ln q}{\partial T}\bigg|_A dT = Nk \ln \frac{T_2}{T_1} = nR \ln \frac{T_2}{T_1}$$