Pop Quiz 2, Chemistry 481, ~~1~~3 Sep 2019. Name: _____

Consider a two-level system, with energy levels $+\varepsilon$ and $-\varepsilon$ (ε is a positive number.

1. Write down an expression for the partition function in terms of $x = \exp(-\varepsilon\beta)$. (2 pts)

$$q = \sum_j \exp(-E_j\beta) = \exp(+\varepsilon\beta) + \exp(-\varepsilon\beta) = (1/x) + x$$

2. Determine an expression for $dx/d\beta$ in terms of x and ε. (2 pts) $x = \exp(-\varepsilon\beta)$ so

$$dx/d\beta = -\varepsilon \exp(-\varepsilon\beta) = -\varepsilon x$$

3. What is the average energy of the two-level system when $T \to \infty$? Justify your answer. (2 pts)
When $T \to \infty$, all levels are equally populated, to $\langle E \rangle$ is the average of all the energies. In this case we have two levels. The average of their energies is just $\langle E \rangle = \frac{1}{2}(-\varepsilon + \varepsilon) = 0$

4. There are large, cold molecular clouds in our galaxy, with temperatures of 10 K and typical number densities of 200 molecules/cm^3. What is the pressure in these clouds? Give your answer in bar. (4 pts) From the ideal gas law $P = nRT/V = Nk_BT/V = k_BT\rho$, where $\rho = N/V$ is the the number density in molecules/m^3. A number density of 200/cm^3 corresponds to
$200 \times 10^6 = 2 \times 10^8$ molecules/m^3. Thus $P = 1.38 \times 10^{-23} \times 10 \times 2 \times 10^8 = 2.76 \times 10^{-14}$ Pa. But 1.01×10^5 Pa = 1 bar. Thus the pressure is
$P = 2.76 \times 10^{-14}$Pa$= 2.76 \times 10^{-14}/1.01 \times 10^5 = 2.74 \times 10^{-19}$ bar.